Now, to study a steady shock flow, one must require the flow variables to have finite constant values at infinity to either side of the shock layer. For finite values of μ, this in turn requires $A = 0$ and gives the result $I + u^2/2 = I_0$, so that the total enthalpy is constant throughout at the value I_0. One now can integrate the momentum equation, replacing the pressure P by its equivalent $(\gamma - 1)\rho I/\gamma$ through the perfect-gas equation of state, where γ is the specific-heat ratio, and expressing I in terms of u^2 and the constant I_0 from the result obtained previously. This finally leads to a relation between u and x only, in which the integration constants must be determined so as to meet the conditions on u at infinity.

In the authors' first reference, the equivalent of Eq. (1) was integrated to obtain Eq. (24) of their first reference. Thus

$$\frac{x}{d} = \frac{k}{C_1 c_p d} \ln\left[1 + \frac{1}{C_3}\left(\frac{u^2}{2} + c_p T\right)\right] \tag{5}$$

which is just a rearranged form of the logarithm of Eq. (4) with μ and c_p each constant and with the integration constant A taken to be unity. Both sides of the preceding equation have been divided by d, which was intended to represent a projectile diameter. In addition to the incorrect association of the velocity u in this equation with half the impact velocity V, the temperature T was replaced by its strong-shock asymptotic value in terms of V.

In the authors' first reference, the constant C_3 and the coefficient of the logarithm on the right in the preceding equation [Eq. (5)] were regarded as parameters to be chosen so as to fit hypervelocity penetration data. The identification of the position variable x with the penetration depth in pellet impact on an infinite target is completely without foundation. After introduction of the square of the stress-wave speed in an elastic bar E/ρ in the authors' first reference, Eq. (5) was finally put into the form shown as Eq. (10) of the note under discussion.

As was remarked earlier, the integration of Eq. (1) is an essential preliminary step to the analysis of a problem to which the authors' Eqs. (3–5) actually apply, namely, that of steady flow through a stationary viscous shock layer in a gas. In that steady flow problem, integration of Eq. (1) simply leads to the conclusion that the total enthalpy of the flow is constant throughout, a fact useful in subsequent analysis in which the final solution for the velocity distribution is required to attain prescribed fixed values at infinity.

In obtaining an integral of Eq. (7) of the note under discussion, the authors again introduce a projectile diameter d by dividing both sides of the resulting equation by that quantity. Thus k_1 in their Eq. (8) contains the factor $1/d$, as does the coefficient of the logarithm in Eq. (5). Again, the authors incorrectly associate the independent position variable x in the one-dimensional, steady flow equations with the penetration depth p in the unsteady impact problem.

Neither the procedure in the note under discussion nor that in their first reference is susceptible to rationalization. The final fit of impact data (the authors' Fig. 5) is indeed surprisingly good, when it is considered that the analysis both in the note under discussion and in their first reference uses, as a starting point, differential equations that do not apply directly even to one-dimensional impact.

In summary, the authors have used an inapplicable theory to study the impact problem, arriving at an analytic expression that was fitted to penetration data by the adjustment of constants in the expression. A steady flow problem, which is associated with one-dimensional impact, and to which their work can actually be considered to apply, has been described in the foregoing discussion. The result of integration of their Eq. (7) possibly may be useful for obtaining an estimate of the thickness of a stationary shock layer in steady flow of a dense viscous material described by a single pressure-density equation, their Eq. (2), provided that the resulting velocity distribution can be made to meet prescribed fixed values at infinity consistent with the values of pressure there through the jump condition for over-all momentum conservation across the transition.

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Reply by Authors to T. A. Zaker

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In the preceding comment on Ref. 1, Zaker questioned the validity of the differential equations of one-dimensional, steady, compressible viscous flow as applied to the hypervelocity impact problem and made specific comments regarding the authors' treatment of the impact problem without, however, offering any suggestions as to how the problem should be treated. First of all, cratering and penetration of targets by projectiles traveling at hypervelocity are very complex physical phenomenon. In the region of hypervelocity impact, the properties of materials involved are beyond the present state of knowledge, and the equation of state for these materials has yet to be established. For these reasons, considerable confusion still exists in the field of hypervelocity impact.

In Ref. 2 a theory of crater formation in solids by hypervelocity impact is studied from the standpoint of radially symmetric advancing shock fronts. The differential equations of one-dimensional, unsteady flow of a compressible inviscid fluid in spherical coordinates lead to a solution based on progressing waves which leads to a $\frac{2}{5}$ power law for penetration vs velocity. Because of the mathematical difficulties that are present, the momentum condition cannot be satisfied by the solution. The results of this elegant analysis are not in good agreement with experimental data. Yuan and Courter[3] have recently analyzed this problem by using the differential equations of an unsteady flow of a viscous compressible fluid in one-dimensional spherical coordinates. Because of the unknown viscous properties of materials at the impact condition, the penetration-impact velocity solution cannot be determined as yet. It is clear that a workable expression for the pentration-impact velocity relation based on unsteady flow theory cannot be realized for sometime to come.

If one considers a system of coordinates moving along with the wave front, the steady flow condition exists in which the respective shocks are stationary. The motion of the fluid in the shock layer is then governed by the equations for one-dimensional steady flow of a viscous compressible fluid [Eqs.

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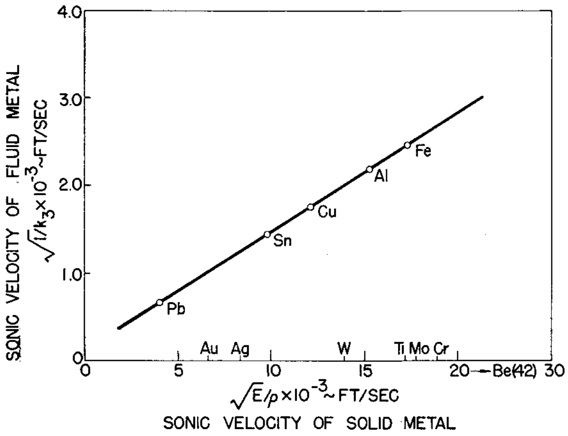

Fig. 1 Sonic velocity of various materials under hypervelocity impact condition and standard condition.

(2–5) of Ref. 1]. The body force is neglected in the foregoing equations [namely, in Eqs. (4) and (5)], but Zaker implies that there exists a body force in the equation of state. The numerical solution of the previous equations would give the velocity distribution in a shock layer (fluid state). Since the material properties at hypervelocity impact are unknown, it is not possible to obtain such a solution at the present time. Equation (8) in Ref. 1 is a terminating solution in which the penetration and impact velocity are related. The authors wish to point out that Zaker's argument regarding the condition on u at $x \to +\infty$ does not apply here because a solid state exists there. Even for a completely gaseous flow, the condition that the flow be uniform at positive infinity is only a necessary, but not sufficient, condition. Hence, the existence of other solutions is physically possible. For air at standard conditions, the thickness of the shock layer at high supersonic speeds, as calculated from Eq. (8), of Ref. 1, is in the order of magnitude of 10^{-5} in. As the temperature of the air increases, the thickness of the shock layer increases considerably.

Zaker remarks that the author's interpretation of the constant $A_2^2 = k_3$ in Eq. (9) of Ref. 1 has no rational physical basis. On the contrary, the significance of the constant k_3 is indicated in Fig. 1. The remarkable relation between the sonic velocities of five different materials at hypervelocity impact conditions and at standard conditions, as shown in Fig. 1, cannot be merely an accident.

The fictitious projectile diameter d, as referred to by Zaker, is introduced in Eq. (9) of Ref. 1 only because the experimental data are given in p/d vs V form. The experimental data used in Ref. 1 are limited to spherical projectiles. If other shapes of projectiles are used, the value of k_1 in Eq. (9) of Ref. 1 would be different. This is similar to the contraction coefficient used in hydraulics, and it can be determined only experimentally. The results given in Ref. 1 are valid for one-dimensional problems of hypervelocity impact where the projectile and target materials are the same. The authors did not claim that these results are applicable to an axially symmetric, three-dimensional impact problem.

As mentioned in Ref. 1, the six constants $A_1, A_2, \ldots, A_6$ in Eq. (8) are very complicated in nature, and they cannot be determined directly until the physical constants of materials at the hypervelocity impact conditions are known. A logical approach to this problem is to evaluate these constants from available experimental data. After neglecting all of the small terms, Eq. (8) in Ref. 1 reduces to Eq. (9). Actually, experimental data were used in reducing Eq. (8) to Eq. (9) by the method of steepest descent with the aid of the CDC 1604 digital computer. This type of iteration technique is well known; hence, Eq. (9) is not a truncated form of Eq. (8) as suggested by Zaker.

In conclusion, the authors wish to restate that the theory of hypervelocity impact is still in the infant stage of development, and more systematic experimental data are needed to aid the advancement of the theory. The analysis given in Ref. 1 is only exploratory in nature, but the results of the analysis may be applied intelligently to a limited number of practical problems. It is well known that the Prandtl mixing-length theory for a turbulent flow lacks rational physical basis, but it is the only practical tool, for the time being, which can be used to solve certain turbulent flow problems. Furthermore, the constants in the universal velocity distribution from Prandtl's theory are determined entirely from experimental data. Finally, with all of the interest, which Zaker has shown in his comment, it would be interesting to know any new theory of hypervelocity impact that he may offer in the near future.

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Comments on "Hypersonic Viscous Flow Near the Stagnation Streamline of A Blunt Body: I. A Test of Local Similarity"

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In a recent paper,[1] Kao has remarked upon the rather unstable nature of certain approximations to the Navier-Stokes equations. The character of this instability is discussed briefly here. The coupled continuity, state, and y-momentum equations [Kao's Eqs. (10) and (12)] may be written, as $y \equiv r - 1 \to 0$ (i.e., approaching stagnation point)

$$V_1'' = B(V_1' - 2U_1) + 0(1) \qquad B \equiv \tfrac{3}{4}(P_1Re/\mu_1V_1) > 0 \tag{1}$$

An identical expression governs stability in the earlier work of Ho and Probstein.[2] With the assumptions $V_1 = \frac{1}{2}V_{1b}''y^2 + 0(y^3)$ and $U_1 = U_{1b}'y + 0(y^2)$ the preceding equation may be perturbed to study the growth of small numerical errors near $y = 0$. The dominant solution is $e^{A/y}$ with $A \equiv -\frac{3}{2}(P_1Re/\mu_1V_1'')_b < 0$, which grows enormously for increasing y. For larger y, the perturbed growth over short intervals is e^{By}, again growing rapidly for increasing y since $B \gtrsim 10$ Re. For decreasing y, the integration step-size h must be controlled to maintain stability according to the criterion $|\partial V''/\partial V'|h = Bh \leq K$, where K is a constant for a given integration scheme. Emanuel[3] lists the values $K = 2.8$ for Kutta's Simpson's rule, and $K = 1.28$ for the fourth-order Adams predictor-corrector.

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